our calculation with those of Vogel [12], and obtains better agreement with the experimental results. Note that experiment shows that these negative line lengths decrease with increasing frequency.

CONCLUSION

A method of calculating quasi-static microstrip discontinuity inductance has been outlined. The computer program written for these calculations has given results which compare favorably with experiment. Curves for some widely used discontinuities have been provided.

ACKNOWLEDGMENT

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The Equivalent Circuit of Some Microstrip Discontinuities

BRIAN EASTER

Abstract—**The experimental characterization of some microstrip structures of common interest, including symmetrical T junctions, is described. Some results are compared with data derived from recent three-dimensional static theory and from the uniform plane-wave model. It is concluded that while the three-dimensional theory requires further improvement, it is generally in much better agreement with the measured data than the two-dimensional uniform plane-wave model.**

I. INTRODUCTION

THERE IS NOW available a substantial body of data on the properties of uniform microstrip lines. In contrast, the circuit designer is often without adequate information on the discontinuity and junction structures comprised in typical practical circuits. Presently available equivalent circuit data fall broadly into two categories. First, there is a growing body of information [1]–[4] on the quasi-static capacitance of microstrip structures, supplemented more recently by studies of the inductance [5]–[7]. In due course this approach should provide accurate quasi-static data on all structures of interest, but, of course, with no indication of dynamic effects. A second source of data derives from the use of a uniform plane-wave parallel plate model of the microstrip cross section as described by Vogel [14] and others, following the method applied by Oliner [8] to symmetrical ("triplate") strip lines: This approach has the great advantage of reducing the problem to two-dimensional complexity. Closed-form expressions are available in classical texts [9] for several quasi-static elements, and some studies [10], [11] have directly tackled the dynamic situation. In addition, reference to Babinet equivalence enables the use of rectangular waveguide data. Against these advantages, there remains the difficulty of any estimation of the error associated with the use of the uniform plane-wave model. It can be noted that not only is the proportion of fringe

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field greater in microstrip than in tri-plate for commonly employed materials and dimensions, but the field decays with distance from the strip much more strongly in the confined tri-plate structure.

We have developed an experimental method of determining microstrip junction parameters which enables a check on the validity of theoretical data and provides data where the theory is not yet fruitful. Some preliminary results [12], [13] have already been presented. We now present more detailed results on corners, T junctions, and crossovers.

II. METHOD OF MEASUREMENT

Useful data on single microstrip discontinuities need to be of high accuracy, desirably better than 1 percent of the characteristic imittance. If conventional measuring procedures and equipment are employed, coaxial-to-microstrip transitions must be used to connect to the structure under test, and even if error correcting procedures are employed, the variability of these transitions is likely to remain a serious limitation. Instead, procedures were devised employing microstrip resonant configurations incorporating the structure to be characterized. This approach has the advantage that only light coupling to the microstrip circuit is required to determine the resonant frequency, and thus errors attributable to the transitions can be much reduced. Evaluation of the equivalent circuit requires accurate knowledge of the phase velocity and of the end effects at the open ends. In addition, because of the moderate Q factor of microstrip, the level of coupling found necessary in practice is associated with a significant perturbation of the resonator and this also must be accurately known. Thus the series gap in microstrip was adapted as a convenient and reproducible means of coupling.

To minimize significant effects due to the variability of substrate properties and the fabrication it is necessary to make a velocity determination for each circuit, either by adding a linear resonator alongside each test configuration, or by etching the test configuration to a suitable form and making a second measurement of the resonant frequency. Careful use of the latter strategy realizes the advantages of a substitution method. Circuits were fabricated on a selected ground and polished 0.995 purity alumina (MRC "Superstrate") 660 ± 2.5 μm thick by directly etching sputtered copper films 3 μm thick. The accuracy of the circuit dimensions was (±0.1 percent ± 5 μm) and effort was directed to minimizing random variations of strip width. For evaluation, individual circuits were measured to an accuracy of ±2 μm.

The end effect l_0 at an open end, and the end effects l_g at a coupling gap were determined using the configuration of Fig. 1, which also defines the equivalent circuit assumed for the gap. l_1 is chosen so that the effective length of this section at the frequency of measurement is $n\lambda_g/2$, where n is an integer and l_g the strip wavelength, while l_2 is chosen to give an effective length of $\lambda_{g/4}$. The free-space wave-

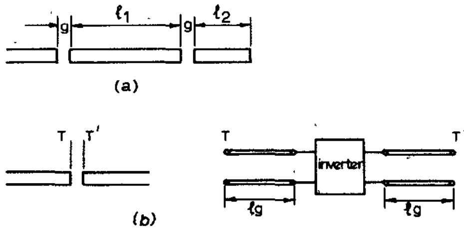


Fig. 1. (a) Resonator configuration for determination of l_0 and l_g. (b) Equivalent circuit of gap.

length for resonance is then given by

$$(l_1 + 2l_g) = \frac{n}{2}\frac{\lambda}{(\epsilon_{\text{eff}})^{1/2}}, \qquad \text{if } l_2 \text{ is present}$$

$$(l_1 + l_0 + l_g) = \frac{n}{2}\frac{\lambda'}{(\epsilon_{\text{eff}})^{1/2}}, \qquad \text{when } l_2 \text{ has been removed}$$

where $\epsilon_{\text{eff}} = (\lambda/\lambda_g)^2$ is the effective dielectric constant.

If two values of l_1 are chosen to give resonance with different n close to the nominal frequency of measurement, then one determination of resonance with l_2 present, together with two determinations with the extra section l_2 removed are sufficient to give ϵ_{eff}, l_0, and l_g. Results of measurements of l_0, l_g at 8 GHz are shown in Fig. 2 and there is excellent agreement with the theoretical values [1], [2] for l_0. Further measurements in the range 5 to 10 GHz have exhibited no evidence of significant variation of l_0, l_g with frequency. The experimental uncertainty of ±10 μm indicated in Fig. 2, largely attributable to substrate variations, corresponds to about $\pm 0.005 Y_0$ at 10 GHz. This represents a significant limitation to the characterization of junction structures using open-ended resonators, and careful choice of test configurations and procedures is necessary to preserve accuracy in the case of the more complex structures. In the following section, a brief outline of the method of measurement, together with measured equivalent circuit data will be presented for three microstrip junction structures.

III. MICROSTRIP JUNCTION MEASUREMENTS

A. Right-Angle Corner

Some results have already been published [12] for this discontinuity. Although the uncompensated corner is of only limited practical interest, further measurements were undertaken in order to provide a better comparison with theoretical data. The measurement employed $\lambda/2$ and λ resonators of L shape as described earlier [12]. The equivalent circuit is defined in Fig. 3, the reference planes being at the inner vertex of the corner as in [12].

In the case of the $\lambda/2$ resonator, there will be a voltage minimum at the corner which will have an effective length l_c, while for the λ resonator, the presence of a voltage maximum will cause the corner susceptance B to be associated with an increase in effective length $(\lambda/\pi)\cdot \tan^{-1}(B/2Y_0)$. Results for three strip widths and a range

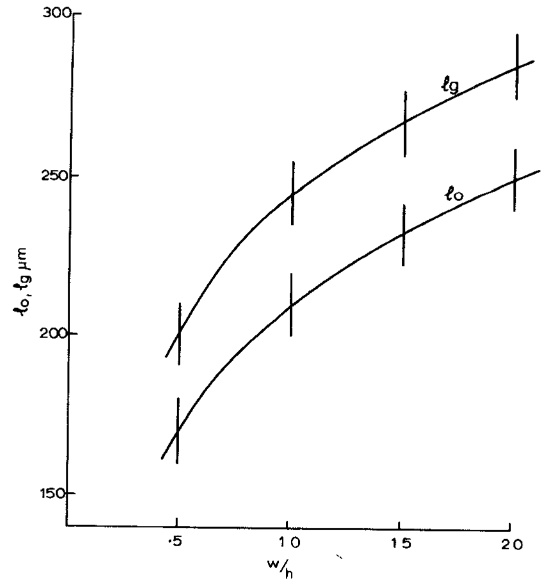


Fig. 2. Measured end effect at an open-end l_0 and at a gap l_g. h = 0.660 mm; g = 0.200 mm; f = 8 GHz; ϵ_r = 9.8.

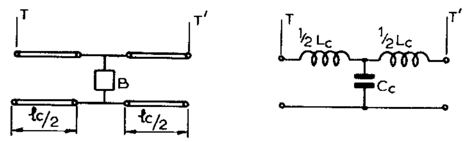


Fig. 3. Right-angle corner: alternative equivalent circuits.

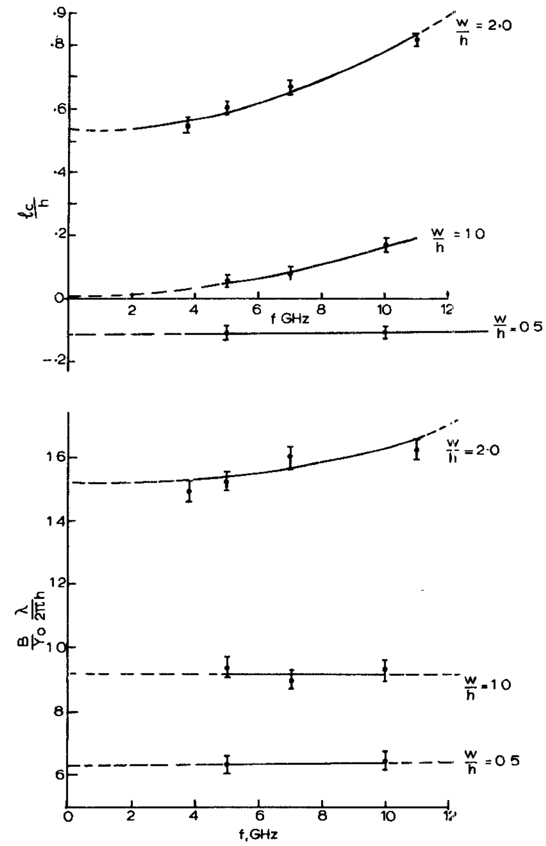


Fig. 4. Right-angle corner: Normalized line length l_c/h and susceptance $(B/Y_0(\lambda/2\pi h)$; h = 0.660 mm; ϵ_r = 9.8.

of frequency are presented in Fig. 4. Significant variation with frequency is observed, particularly for the wider strips. In order to provide a better comparison with quasi-static theory, the results were extrapolated to zero frequency assuming a variation proportional to the square of frequency. Table I presents the extrapolated measured data, theoretical data from three-dimensional studies [4], [7], and data calculated from the uniform plane-wave model for comparison.

The uniform plane-wave model followed the definition given by Leighton and Milnes [15], the circuit elements being determined from the precise static theory [9]. All the data were, of course, adjusted to the same reference planes.

B. Microstrip T Junction

The T junction is of practical importance, occurring frequently and often having equivalent circuit elements of significant magnitude. Measurements have been restricted to symmetrical forms with a "through" arm impedance of approximately 50 Ω and for a range of stub arm impedance. Fig. 5 shows the equivalent circuit and reference planes assumed and also shows the measurement circuit configurations. The circuit of Fig. 5(b) enables an accurate determination of the line length l_a by determining the resonance of the λ/2 through arm before and after removing the stub. The measurement is dependent essentially on the ability to remove the stub accurately and cleanly, and on the repeatability of the resonance

TABLE I
ZERO-FREQUENCY CORNER DATA

$\ell_c/h = \frac{L_c}{L_\infty h}$ where L_∞ is the distributed inductance of uniform microstrip

w/h	Extrapolated Measurements	Thompson & Gopinath (7)	uniform plane wave model
0.5	−0.115 ± .015	− 0.25	− 0.491
1.0	+0.010 ± .015	− 0.05	− 0.310
2.0	+0.535 ± .015	+ 0.495	+ 0.137

$C_c/C_\infty h$ where C_∞ is the distributed capacitance of uniform microstrip

w/h	Extrapolated Measurement	Benedek & Silvester (4)	uniform plane wave model
0.5	0.520 ± .030	0.397	0.5
1.0	0.930 ± .030	0.906	1.0
2.0	2.050 ± .030	1.916	2.0

Note: ϵ_r = 9.8.

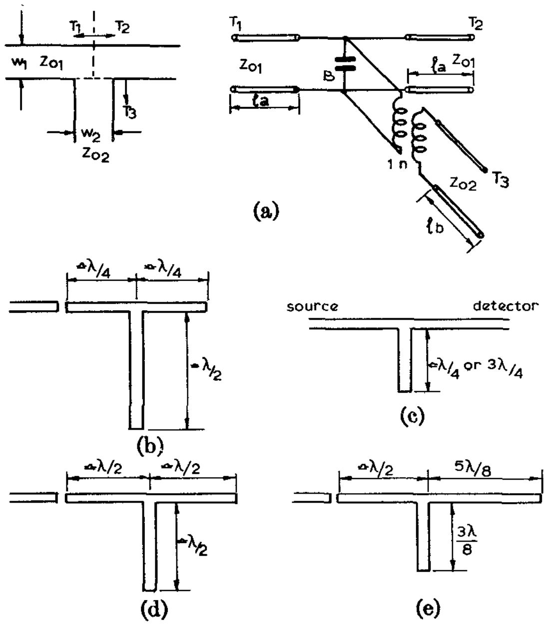


Fig. 5. T junction. (a) Equivalent circuit. (b)–(e) Configurations to determine l_a, l_b, B, and n, respectively.

determination. Data for l_a with a spread of a few micrometers were typical. l_b may be obtained from the frequency of peak attenuation with the stub arm of Fig. 5(c) an odd number of quarter-wavelengths, and will be directly dependent on l_0, the assumed end effect of the open end of the stub.

The configuration of Fig. 5(d) enables B to be determined. With careful design, an accurate knowledge of n will not be required, but in practice this may entail a preliminary attempt in order to get the stub sufficiently close to the half-wave length. It might appear that the value for B would be subject to an uncertainty corresponding to that of $(2l_0 + l_g + l_b + 2l_a)$. However, if the stub is etched off and the circuit remeasured to obtain ϵ_{eff} from the λ resonance of the through arm, it may be shown that there is some cancellation in the uncertainty due to the assumed value of $(l_0 + l_g)$. It may also be noted that l_b is directly dependent on the value of l_0 and the uncertainty in l_a is relatively small. As a result, the error range for $(B/Y_0)(\lambda/2\pi)$ is only slightly greater than that for l_0.

The impedance ratio n^2 is determined by the rather similar configuration of Fig. 5(e). The result is dependent on the elements already determined l_0, l_g, l_a, l_b, and B, and some further reduction of accuracy must be accepted. This may be alleviated by again using the resonance of the through arm with the stub removed, and the accuracy of determination of n^2 is estimated as ± 2 percent.

Measured data are presented in Figs. 6 and 7 while extrapolated zero-frequency values are compared with theoretical values in Table II.

C. Crossover Junction

This four-arm junction can result from the requirement for a stub of low impedance, such that a single stub is of excessive width, where a possible solution is to employ two

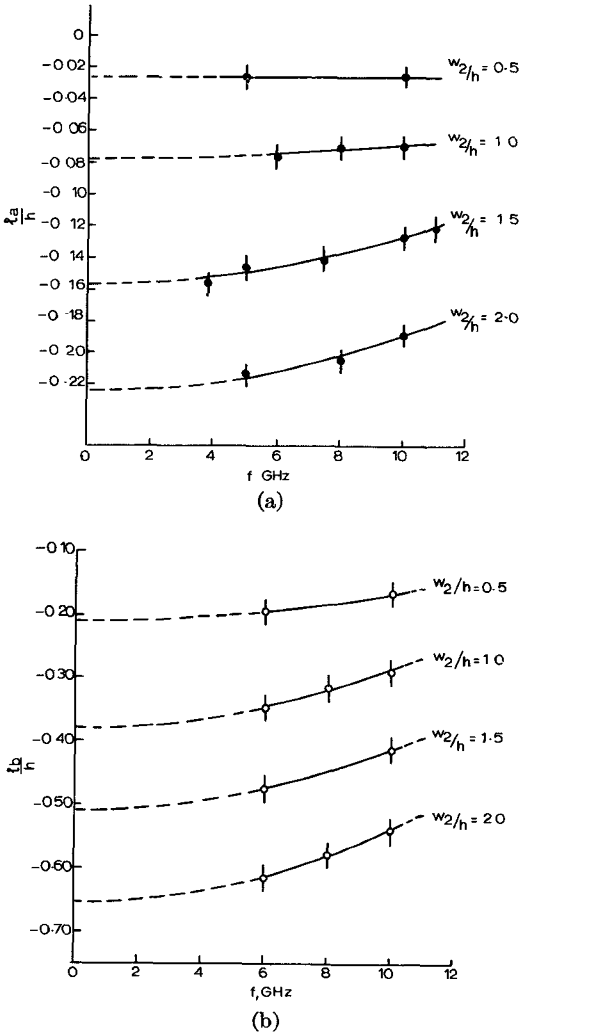
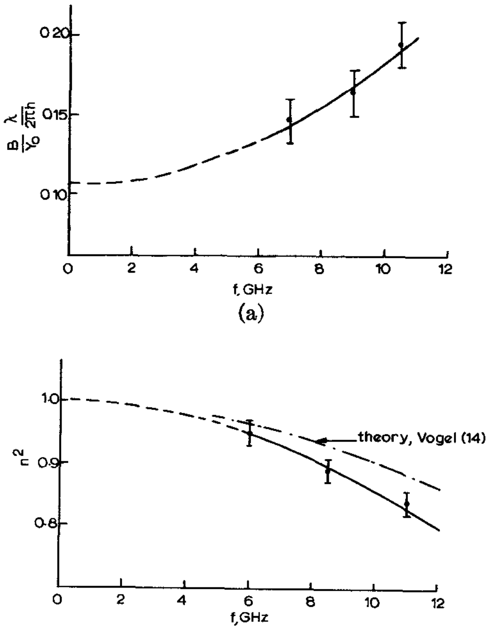


Fig. 6. T junction. (a) Normalized line length l_a/h. (b) Normalized line length l_b/h. $W_1/h = 1.0$; $h = 0.660$ mm; $\epsilon_r = 9.8$.



(b)

Fig. 7. T junction. (a) Normalized susceptance $(B/Y_0)(\lambda/2\pi h)$. (b) Impedance ratio n^2. $W_1/h = W_2/h = 1.0$; $h = 0.660$ mm; $\epsilon_r = 9.8$.

TABLE II
ZERO-FREQUENCY T JUNCTION DATA

ℓ_a/h			
W_2/h	Extrapolated Measurements	Thompson & Gopinath (7)	uniform plane wave model
0.5	- 0.026 ± .010	- 0.021	- 0.132
1.0	- 0.078 ± .010	- 0.068	- 0.227
1.5	- 0.156 ± .010	- 0.144	- 0.331
2.0	- 0.224 ± .010	- 0.212	- 0.444
ℓ_b/h			
W_2/h	Extrapolated Measurement	Thompson & Gopinath (7)	uniform plane wave model
0.5	- 0.210 ± .02	- 0.320	- 0.549
1.0	- 0.380 ± .02	- 0.485	- 0.657
1.5	- 0.510 ± .02	- 0.635	- 0.793
2.0	- 0.655 ± .02	- 0.840	- 0.946
$C_T/C_\infty h$	where C_T is the total capacitance of the junction including the line lengths ℓ_a, ℓ_b; C_∞ is the distributed capacitance of uniform microstrip		
W_2/h	Extrapolated Measurement	Benedek & Silvester (4)	uniform plane wave model
1.0	- 0.425 ± .03	- 0.614	- .984

Note: $W_1/h = 1$, $\epsilon_r = 9.8$.

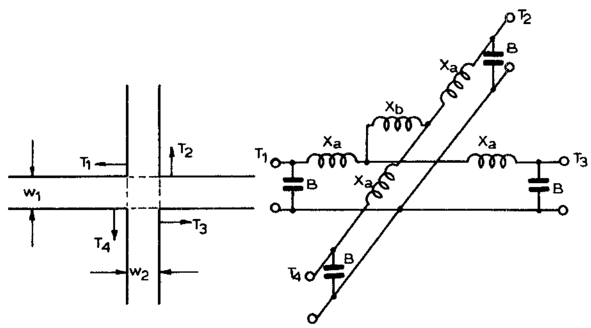


Fig. 8. Equivalent circuit for microstrip crossover. $h = 0.660$ mm; $W_1/h = W_2/h = 1.0$; $f = 7$ GHz; $\epsilon_r = 9.8$.

$$\frac{Xa}{Z_0}\frac{\lambda}{2\pi h} = 0.365 \pm 0.015$$

$$\frac{Xb}{Z_0}\frac{\lambda}{2\pi h} = -0.695 \pm 0.015$$

$$\frac{B}{Y_0}\frac{\lambda}{2\pi h} = 0.091 \pm 0.010.$$

stubs in parallel, connected on either side of the through line. The stub length requires correction for the effect of the equivalent circuit of the junction, and the question arises as to whether the arrangement can, indeed, be strictly regarded as a straightforward parallel connection.

While only one example of a cross has been characterized, the results are of some interest. Fig. 8 gives the equivalent circuit of an equal arm symmetrical 90° cross at

7 GHz on 0.660-mm alumina, the lines being of approximately 50-Ω Z_0. Alternative equivalent circuits can be drawn, for example, with the center inductor incorporated as mutual inductance between either or both pairs of opposite branches. The elements of the circuit were determined as indicated in Fig. 5(b), (c), and (d) for the T junction, and taking advantage of the symmetry of the junction. If one pair of opposite branches are regarded as the through line, and the other pair used as stubs, it is clear that there is significant interaction between the two stubs. Thus if one stub presents a high impedance at the reference plane the other stub will behave very similarly to one connected at a simple T junction, the corresponding value of l_b/h being -0.335. However, if the stubs present similar impedances the effective value of l_b/h is -1.025. Thus strictly, one can only regard the situation as a parallel connection of two stubs if the stubs are identical and the appropriate line length correction is used. While there is not sufficient data to permit extrapolation to zero frequency, the value $+0.091$ for $(B/Y_0)(\lambda/2\pi h)$ may be compared with -0.050 calculated from the results of [4] and -0.242 for the uniform plane-wave model.

IV. CONCLUSIONS

In general, the results of the three-dimensional static theory [4], [7] are in much better agreement with the measurements than data derived from the uniform plane-wave model. An inevitable conclusion would appear to be that at least for the small w/h values associated with alumina microstrip, the two-dimensional uniform plane-wave model is a very unreliable basis for predicting the behavior of microstrip junctions. Indeed, in some cases it would be better to ignore the presence of a circuit element than to use this prediction of its value. However, it is also to be noted that the discrepancy between measured results and the three-dimensional static theory needs to be reduced appreciably before some results of the latter can be used for accurate design work with confidence. In addition, there is evidence of sufficient variation of circuit element values with frequency to indicate that unmodified quasi-static data will continue to have limitations with regard to the wider linewidths at the higher frequencies.

Clearly the volume of data required to cover most of the possible requirements of a circuit designer is considerable. In the absence of a comprehensive dynamic theory of sufficient accuracy, a possible solution is to concentrate on improving the three-dimensional static theory and to use the results of representative measurements, such as those described here, as a guide to the frequency dependence to be expected.

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